Predicting the Λ binding energy in nuclear matter

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Abstract

The purpose of this note is to report predictions of the binding energy of the Λ hyperon in nuclear matter using the latest version of the Jülich nucleon-hyperon meson-exchange potential. Results from a conventional Brueckner calculation are compared with previously reported values. A calculation including Dirac effects on the Λ single-particle potential is also presented. Issues encountered in Dirac calculations with nucleon-hyperon potentials are discussed.

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1 Introduction

There are important motivations for including strange baryons in nuclear matter. The presence of hyperons in stellar matter tends to soften the equation of state (EoS), with the consequence that the predicted neutron star maximum masses become considerably smaller. With recent constraints allowing maximum masses larger than previously accepted limits [1], accurate microscopic calculations which include strangeness (in addition to other important effects, such as those originating from relativity), become especially important and timely. Furthermore, as far as terrestrial nuclear physics is concerned, studies of hyperon energies in nuclear matter naturally complement our knowledge of hypernuclei.

Microscopic calculations of nuclear matter properties have been reported earlier within the non-relativistic Brueckner-Hartree-Fock framework (see, for instance, Refs. [2, 3]), using the Nijmegen [4] and/or the Jülich [5, 6] nucleon-hyperon (NY) meson-exchange potentials.

In this work we use the most recent meson-exchange NY potential from the Jülich group [7]. Given that there are significant differences between this and the previous (energy independent) version of the Jülich NY potential [6], it will be interesting to see how those differences reflect onto G-matrix calculations. Before moving on to our final objective, which is a microscopic determination of the EoS for hyperonic matter, we will first confront a much simpler scenario, namely the one of nuclear matter at some Fermi momentum k_F^N in the presence of a "Λ impurity". This calculation, the outcome of which is the binding energy of a Λ hyperon in nuclear matter, will allow us to explore the behavior of the new NY potential in nuclear matter before addressing more involved situations. Some empirical information is available for the

Λ binding energy from analyses of (π, K) and (K, π) reactions and studies of energies of hypernuclei [8].

We will also report on our prediction for the Λ binding energy from a Dirac-Brueckner-Hartree-Fock (DBHF) calculation. To the best of our knowledge, such calculation has not been performed before.

2 Calculations using the non-relativistic Brueckner G-matrix

For matter with non-vanishing hyperonic density, the nucleon, Λ, and Σ single-particle potentials are the solution of a coupled self-consistency problem, which reads, schematically

$$\begin{aligned} U_N &= \int_{k<k_F^N} G_{NN} + \int_{k<k_F^\Lambda} G_{N\Lambda} \int_{k<k_F^\Sigma} G_{N\Sigma} \\ U_\Lambda &= \int_{k<k_F^N} G_{\Lambda N} + \int_{k<k_F^\Lambda} G_{\Lambda\Lambda} + \int_{k<k_F^\Sigma} G_{\Lambda\Sigma} \\ U_\Sigma &= \int_{k<k_F^N} G_{\Sigma N} + \int_{k<k_F^\Lambda} G_{\Sigma\Lambda} + \int_{k<k_F^\Sigma} G_{\Sigma\Sigma} \end{aligned} \tag{1}$$

In the equations above, G_{NN}, G_{NY}, and $G_{YY'}$, $(Y, Y' = \Lambda, \Sigma)$, are the nucleon-nucleon, nucleon-hyperon, and hyperon-hyperon G-matrices at some nucleon and hyperon densities defined by the Fermi momenta k_F^N and k_F^Y.

Following the earlier calculation from Ref. [6], which we want to compare with, we make the following assumptions and approximations:

- We consider the case of symmetric nuclear matter at some Fermi momentum k_F^N in the presence of a "Λ impurity", i.e. $k_F^\Lambda \approx 0$.
- We take the single-nucleon potential from a separate calculation of symmetric matter [9]. Notice that the Λ potential is quite insensitive to the choice of U_N, as reported in Ref. [6] and as we have observed as well.
- Both Λ and Σ are included in the coupled-channel calculation of the NY G-matrix, but free-space energies are used for the latter.

The parameters of the Λ potential, on the other hand, are calculated self-consistently with the G_{NY} interaction, which is the solution of the Bethe-Goldstone equation with one-boson exchange NY potentials. In the Brueckner calculation, density-dependent effects come in through angle-averaged Pauli blocking and dispersion.

For some NY initial state, the starting energy that enters in the scattering equation is

$$E_0 = e_\Lambda(p_\Lambda) + e_N(p_N). \tag{2}$$

For nucleons and Λ's, we write the single-particle energy, in the non-relativistic case, as

$$e_i(p) = \frac{p_i^2}{2m_i} + U_i(p_i) + m_i \tag{3}$$

$(i = N, \Lambda)$. (Angle-averaged) Pauli blocking is applied to all intermediate two-baryon states. The integration in the Bethe-Goldstone equation is handled using standard methods to eliminate the angular dependence [6]. We adopt the continuous choice for the single-particle potential. Once the latter is obtained, the value of $-U_\Lambda(p_\Lambda)$ at p_Λ=0 provides the Λ binding energy in nuclear matter, B_Λ.

In this work, we will apply the latest meson-exchange model by the Jülich group [7], which will be denoted by NY05. As shown and discussed extensively in Ref. [7], there are several remarkable differences between this model and the previous *NY* Jülich potential [6] (NY94). The main new feature of NY05 is a microscopic model of correlated $\pi\pi$ and $K\bar{K}$ exchange to constrain both the σ and ρ contributions [7]. The usual one-boson-exchange contributions from the lowest pseudoscalar and vector meson multiplets are also present, with the coupling constants determined by SU(6). This makes the long and intermediate range parts of the potential well determined. New short-range features are included through the $a_0(980)$ meson and a strange scalar meson with a mass of approximately 1000 MeV. Both of these are parametrized phenomenologically in terms of one-boson exchanges in the respective spin-isospin channels. The model describes well the available data on integrated as well as differential cross sections [7]. Also, the hypertriton binding energy is well reproduced [10].

There are, though, some major quantitative differences between NY05 and NY94 in specific partial waves. Most noticeably, the new model predicts a considerably larger scattering length in the singlet channel. These differences turn out to have a large impact on in-medium predictions. With the new model, we obtain considerably more attraction than Reuber *et al.* [6], approximately 50 MeV at k_F^N=1.35 fm^{-1} for B_Λ, rather than 30 MeV. A value of 49.7 MeV has been reported by H. Polinder with NY05 [11]. Notice that a value of 30 MeV is generally accepted as the "empirical" one [8], which opens some interesting questions:

- The value predicted by NY05 for the hypertriton binding energy is 2.27 MeV, in satisfactory agreement with the experimental value of 2.354(50) MeV. How reliable is the "empirical" value for the Λ binding energy in nuclear matter?
- How will the additional attraction impact the EoS and neutron star predictions? How will those predictions compare to the most recent constraints?

To better highlight the potential model dependence of the predicted Λ binding energy, we show in Table 1 how selected partial waves contribute to it. For each partial wave, column NY94 shows the results given in Ref. [11], whereas column NY05 are the predictions from this work. Clearly the largest contribution to the

Table 1: Contributions to the Λ binding energy from selected partial waves as obtained in non-relativistic Brueckner-Hartee-Fock calculations. NY05 and NY94 indicate the Jülich NY potentials from Ref. [7] and Ref. [6], respectively.

Partial wave	NY94, BHF	NY05, BHF
$^1S_0$	3.6	8.73
$^3S_1$+$^3D_1$	27.2	37.69
$^3P_0$	-0.6	0.69
$^1P_1$+$^3P_1$	-2.0	0.13
$^3P_2$+$^3F_2$	0.8	3.27
Total (all states)	29.8	51.27

model dependence originates from the S-waves, although the contribution from the P-waves is also dramatically different between the two sets of predictions (but much smaller than the one from the S-waves).

The NY05 entries agree well with those from Ref. [11] for the same potential; the relatively minor differences are possibly due to different choices for the single-particle spectrum, the handling of the angular dependence in the scattering equation, and, to a very small extent, the choice of the nucleon potential.

3 Dirac effects on the Λ binding energy

The relation between the non-relativistic Brueckner approach and the relativistic framework (known as Dirac-Brueckner-Hartree-Fock, DBHF) has been discussed for a long time. Already in Ref. [12] it was shown how relativistic effects tie in with virtual excitations of pair terms. Lately, these concepts have been revisited in more detail [13] and with similar conclusions. In short, the Dirac effect on the EoS of nucleonic matter is an essential saturating, and strongly density dependent, mechanism, which effectively accounts for the class of three-body forces originating from virtual nucleon-antinucleon excitations. When hyperon degrees of freedom are included, for reasons of consistency, those should then be subjected to the same correction.

We have incorporated DBHF effects in the present calculation, which amounts to involving the Λ single-particle Dirac wave function in the self-consistent calculation through the Λ effective mass, m^*_Λ. Similarly to what is done for nucleons [14], we fit the single-particle energy for Λ's using the ansatz

$$e_\Lambda(p) = \sqrt{(m^*_\Lambda)^2 + p^2} + U_V^\Lambda \tag{4}$$

with $m^*_\Lambda = m_\Lambda + U_S^\Lambda$, and U_S^Λ and U_V^Λ the scalar and vector potentials of the Λ baryon.

A problem with the Jülich NY potential in conjunction with DBHF calculations is the use of the pseudoscalar coupling for the interactions of pseudoscalar mesons

(pions and kaons) with nucleons and hyperons. For the reasons mentioned above (that is, the close relationship between Dirac effects and "Z-diagram" contributions), this relativistic correction is known to become unreasonably large when applied to a vertex involving pseudoscalar coupling. On the other hand, the gradient (pseudovector) coupling (also supported by chiral symmetry arguments) largely suppresses antiparticle contributions. To resolve this problem, one can make use of the on-shell equivalence between the pseudoscalar and the pseudovector coupling, which amounts to relating the coupling constants as follows:

$$g_{ps} = f_{pv} \frac{m_i + m_j}{m_{ps}}, \tag{5}$$

where g_{ps} denotes the pseudoscalar coupling constant and f_{pv} the pseudovector one; m_{ps}, m_i, and m_j are the masses of the pseudoscalar meson and the two baryons involved in the vertex. This procedure can be made plausible by writing down the appropriate one-boson-exchange amplitudes and observing that, redefining the coupling constants as above, we have (see Ref. [14] for the two-nucleon case)

$$V_{pv} = V_{ps} + ... \tag{6}$$

where the ellipsis stands for off-shell contributions. Thus, the pseudoscalar coupling can be interpreted as pseudovector coupling where the off-shell terms are ignored. This is what we apply in our DBHF calculations.

In the coupled channel calculation, evaluation of $G_{N\Lambda}$ involves the transition potentials $V_{N\Lambda\to N\Lambda}$, $V_{N\Lambda\leftrightarrow N\Sigma}$, and $V_{N\Sigma\to N\Sigma}$ (all with total channel isospin equal to 1/2), plus the corresponding exchange diagrams. Because in the present scenario (of a Λ impurity in nucleonic matter) the Σ hyperon is not given an effective mass, Dirac effects are applied only in $V_{N\Lambda\to N\Lambda}$. A diagram where not all of the baryon lines are Dirac-modified may yield a Dirac effect that is artificially skewed. Moreover, since we find that the net contribution to the Λ binding energy from the coupling to the $N\Sigma$ channels is rather small (≈ 1.3 MeV), we anticipate Dirac effects from those channels to be negligibly small.

Finally, a comment is in place concerning meson propagators. In standard DBHF calculations [14], the so-called Thompson equation (a relativistic three-dimensional reduction of the Bethe-Salpeter equation) is used for two-baryon scattering. In the Thompson formalism, static propagators are employed for meson exchange, i.e.

$$-\frac{1}{m_\alpha^2 + (\vec{q}\,' - \vec{q})^2} \tag{7}$$

where m_α denotes the mass of the exchanged meson and $\vec{q}, \vec{q}\,'$ are the baryon momenta in their center-of-mass frame before and after scattering. The Jülich NY potentials are based upon time-ordered perturbation theory [5] and use a meson propagator given by

$$\frac{1}{\omega_\alpha(z - E_i - E_j - \omega_\alpha)} \tag{8}$$

Table 2: Contributions to the Λ binding energy from selected partial waves obtained with the Jülich NY05 potential in a DBHF calculation.

Partial wave	NY05, DBHF
$^1S_0$	8.14
$^3S_1$+$^3D_1$	36.45
$^3P_0$	0.07
$^1P_1$+$^3P_1$	-1.19
$^3P_2$+$^3F_2$	3.21
Total (all states)	47.4

with $\omega_\alpha = \sqrt{m_\alpha^2 + (\vec{q}' - \vec{q})^2}$; $E_i = \sqrt{m_i^2 + \vec{q}'^2}$ and $E_j = \sqrt{m_j^2 + \vec{q}^2}$ are baryon energies and z is the starting energy of the two-baryon system. In order to eliminate the energy dependence, Reuber *et al.* [6] replaced the original z with

$$z = \frac{1}{2}(m_1 + m_2 + m_3 + m_4), \tag{9}$$

where the m_i's denote the baryon masses of the four legs in the one-meson exchange diagram. In any case, the Jülich meson propagator involves the baryon masses. Replacement of these free-space masses with in-medium values would create medium effects on meson propagation which we do not wish to include in our nuclear matter calculations. The reason for keeping free-space masses in the meson propagator is twofold. First, standard DBHF calculations do not include medium effects on meson propagation as they typically use Eq. (7), which does not not depend on baryon masses. Second, medium effects on meson propagation constitute a separate class of effects that we are not concerned with in the present context and are typically not perceived as part of the DBHF approach.

Having taken the steps described above, we proceed to the DBHF calculation and find a moderate reduction of B_Λ, by approximately 4 MeV, due to Dirac effects. This is roughly 50% of the corresponding effect on the nucleon potential. In Table II, contributions from selected partial waves are again shown. A large part of the effect can be attributed to increased repulsion in S and P waves, especially $^3P_1$.

4 Conclusions

We have reported on non-relativistic and Dirac-Brueckner-Hartree-Fock predictions of the Λ binding energy in nuclear matter at normal nuclear density. The magnitude of the Dirac effect is approximately 1/2 of the corresponding effect on the binding of a nucleon in nuclear matter.

We also noticed and discussed the remarkably different predictions of this "observable" as obtained using the 2005 or the 1994 versions of the Jülich NY potentials. Given that there are noticeable differences between the two free-space potentials, some potential model dependence is to be expected. In this case, though, it would be appropriate to say that those free-space differences are considerably "amplified" in the nuclear matter calculation. Notice that this comparison was done between predictions as obtained from conventional (BHF) calculations. But the conclusions would be unimpacted by the Dirac effect, which is *much* smaller than the differences originating from the use of the two potential models.

The natural extension of this preliminary calculation will be a fully self-consistent DBHF calculation of U_N, U_Λ, and U_Σ for diverse N, Λ and Σ concentrations. Such work is in progress.

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